THE INFLUENCE OF MODIFIED GRAVITY ON THE LATE-TIME ACCELERATION OF THE UNIVERSE

Bharvad Sweta Chhanabhai[1], Dr. Ashok Patel[2]

[1] Ph.D. Research Scholar, Monark University, Ahmedabad, Gujarat
[2] Assistant Professor, Monark University, Ahmedabad, Gujarat

ABSTRACT

The accelerated expansion of the universe, first discovered through observations of distant Type Ia supernovae, poses one of the most profound challenges to our current understanding of cosmology and gravitation. While the standard ΛCDM model addresses this phenomenon by invoking a cosmological constant or dark energy component, it suffers from theoretical issues such as fine-tuning and the coincidence problem. In response, modified theories of gravity have emerged as alternative frameworks that aim to explain late-time cosmic acceleration through extensions of Einstein's General Relativity. This paper explores the influence of modified gravity on the late-time acceleration of the universe by examining several theoretical models, including f(R) gravity, scalar-tensor theories, and f(T) gravity. The mathematical formulations of these theories are presented alongside their cosmological solutions, followed by an analysis of observational constraints drawn from supernovae, cosmic microwave background anisotropies, large-scale structure data, and gravitational wave observations. Case studies and comparative analyses highlight the strengths and limitations of each model in replicating observational data. The study concludes that while several modified gravity models provide viable alternatives to dark energy, their compatibility with the full spectrum of cosmological and astrophysical data remains an active area of research. The ongoing and upcoming high-precision cosmological surveys are expected to play a critical role in validating or refuting these theoretical approaches.

KEYWORDS: Modified Gravity, Cosmic Acceleration, f(R) Gravity, Scalar-Tensor Theory, f(T) Gravity, Dark Energy, General Relativity, Cosmological Observations, Structure Formation, Gravitational Waves

1. INTRODUCTION

The discovery of the accelerated expansion of the universe at the end of the 20th century marked a turning point in modern cosmology. Observations of distant Type Ia supernovae, coupled with data from the cosmic microwave background (CMB) and large-scale structure surveys, revealed that not only is the universe expanding, but it is doing so at an ever-increasing rate. This unexpected finding posed a fundamental challenge to the classical understanding of cosmology governed by Einstein's General Theory of Relativity (GR). Within the framework of GR, this acceleration is usually attributed to a mysterious and dominant component known as dark energy, which constitutes nearly 70% of the energy content of the universe.

While the ΛCDM (Lambda Cold Dark Matter) model, which incorporates the cosmological constant (Λ) as a representation of dark energy, has been remarkably successful in describing many cosmological observations, it is not without its theoretical problems. The cosmological constant problem—arising from the vast discrepancy between theoretical predictions and observed values—and the coincidence problem—questioning why the densities of dark energy and matter are of the same order precisely today—have led scientists to seek alternative explanations. These challenges suggest that the standard model may be incomplete and that new physics could be needed to understand the true nature of cosmic acceleration.

One promising line of inquiry is the modification of gravitational theory itself. Rather than introducing an unknown form of energy to account for the observed acceleration, some researchers propose altering the laws of gravity at large scales. Modified theories of gravity aim to explain the universe's late-time acceleration without invoking dark energy, offering a more unified and potentially natural explanation. These theories modify Einstein's field equations, often by adding new terms, extra dimensions, or scalar fields, thereby changing the dynamics of cosmic expansion.

Several models have been proposed under the umbrella of modified gravity, including f(R) gravity, f(T) gravity, scalar-tensor theories such as the Brans–Dicke model, and braneworld scenarios like the Dvali–Gabadadze–Porrati (DGP) model. Each of these approaches offers a unique mechanism by which the universe could accelerate in the absence of dark energy. For example, in f(R) gravity, the Ricci scalar RR in the Einstein-Hilbert action is replaced with a more general function, allowing for richer dynamics that can mimic dark energy-like behavior. Similarly, scalar-tensor theories introduce scalar fields that interact with gravity, altering the effective gravitational strength over time.

This study focuses on examining the influence of these modified gravity theories on the late-time acceleration of the universe. It seeks to understand whether these models can

not only reproduce the observed expansion history but also align with other cosmological observations and remain free from theoretical inconsistencies. By comparing different modified gravity frameworks and analysing their cosmological implications, this study contributes to the ongoing quest for a deeper and more complete understanding of the universe's large-scale behaviour.

2. COSMOLOGICAL ACCELERATION AND THE STANDARD MODEL

The discovery that the universe is undergoing an accelerated expansion was a revolutionary moment in cosmology. This phenomenon was first revealed in the late 1990s through independent observations by the Supernova Cosmology Project and the High-Z Supernova Search Team. By studying Type Ia supernovae—stellar explosions that serve as standard candles due to their consistent intrinsic brightness—astronomers concluded that distant galaxies were receding at a faster rate than could be explained by a decelerating universe. This contradicted the long-standing assumption that the gravitational pull of matter would slow down the cosmic expansion over time.

To explain this surprising acceleration within the framework of Einstein's General Relativity, cosmologists introduced the concept of dark energy, a hypothetical form of energy with a strong negative pressure that drives the acceleration. The most widely accepted model incorporating this idea is the Lambda Cold Dark Matter (ΛCDM) model, where the cosmological constant (Λ) represents the energy density of empty space. According to this model, the energy content of the universe consists of approximately 70% dark energy, 25% dark matter, and 5% ordinary (baryonic) matter. The ΛCDM model has become the standard cosmological model due to its consistency with a wide range of observational data, including the cosmic microwave background radiation (as measured by WMAP and Planck), baryon acoustic oscillations, and large-scale galaxy distribution.

Despite its empirical success, the ΛCDM model is not without significant theoretical issues. One of the most prominent challenges is the cosmological constant problem, which arises from quantum field theory predictions of vacuum energy density being up to 120 orders of magnitude larger than the observed value of Λ. This enormous discrepancy highlights a profound gap in our understanding of fundamental physics. Additionally, the coincidence problem questions why the density of dark energy is comparable to that of matter precisely in the current epoch, suggesting a fine-tuning that many physicists find unsatisfying.

These limitations have led researchers to consider whether the late-time acceleration could be due not to an exotic energy component, but to a breakdown of General Relativity at cosmic scales. This line of reasoning forms the foundation for modified gravity theories, which aim to explain cosmic acceleration by altering the geometric framework of gravity itself rather than introducing new forms of energy. In doing so, they seek to preserve the successes of General Relativity at local scales while providing a more complete description of the universe's expansion history.

Thus, while the standard cosmological model remains the dominant framework in modern cosmology, its reliance on a poorly understood component—dark energy—has motivated the exploration of alternative gravitational theories. These efforts aim to reconcile theoretical inconsistencies and offer new insights into the fundamental forces that govern the cosmos.

3. MODIFIED GRAVITY THEORIES

The pursuit of a deeper understanding of the universe's accelerated expansion has led to the development of modified gravity theories, which offer a compelling alternative to dark energy-based explanations. Instead of introducing an unknown energy component with repulsive gravity, these theories propose that the laws of gravity themselves may require modification on cosmological scales. The foundation for this idea lies in the fact that Einstein's General Theory of Relativity, although extremely successful in describing gravitational phenomena on solar system and galactic scales, might not fully capture the behaviour of gravity at the largest scales of the universe.

Modified gravity theories attempt to address the observed late-time acceleration by extending or generalizing the Einstein-Hilbert action, from which Einstein's field equations are derived. These modifications often involve changing the dependence of the action on curvature terms or introducing additional fields that interact with gravity. One of the simplest and most widely studied extensions is f(R) gravity, where the Ricci scalar RR in the Einstein-Hilbert action is replaced with a general function f(R). This approach modifies the field equations and leads to a richer set of cosmological dynamics that can reproduce acceleration without a cosmological constant. Various forms of f(R) have been proposed and tested against observational data, some of which exhibit behaviour similar to dark energy at late times.

Another notable class of theories is scalar-tensor gravity, which includes an additional scalar field that mediates gravitational interactions along with the metric tensor. The most well-known of these is the Brans–Dicke theory, which introduces a time-varying gravitational "constant" determined by a scalar field. More general scalar-tensor theories have been developed within the framework of Horndeski gravity, the most general class of scalar-tensor theories that yields second-order field equations and avoids instabilities. These models can produce accelerated expansion and are compatible with many existing observational constraints.

Teleparallel gravity represents a fundamentally different approach. In General Relativity, gravity is described as the curvature of spacetime. In contrast, f(T) gravity formulates gravity in terms of torsion rather than curvature. Here, the torsion scalar T replaces the Ricci scalar RR in the action, and modifying this to a function f(T) leads to new dynamics that can also account for late-time cosmic acceleration. f(T) gravity has attracted attention for its mathematical simplicity and potential

to naturally explain cosmic acceleration without dark energy.

Other intriguing frameworks include braneworld models, such as the Dvali–Gabadadze–Porrati (DGP) model. In this theory, our four-dimensional universe is envisioned as a "brane" embedded in a higher-dimensional bulk. Gravity can leak into the extra dimension at large distances, effectively weakening its influence and giving rise to acceleration. While elegant in conception, some braneworld models face challenges such as ghost instabilities and compatibility with precision cosmological data. Each of these modified gravity theories provides unique insights into the nature of gravity and the large-scale dynamics of the universe. However, they must pass stringent theoretical and observational tests to be considered viable. These include consistency with solar system experiments, the stability of cosmological solutions, and compatibility with high-precision observational data such as supernovae measurements, the cosmic microwave background, and structure formation.

Overall, modified gravity theories offer a rich and promising framework to understand cosmic acceleration without invoking dark energy. They represent a bold attempt to extend our understanding of gravity beyond Einstein's theory and may ultimately lead to a more unified and complete description of the fundamental forces shaping the cosmos.

4. MATHEMATICAL FORMULATION AND COSMOLOGICAL SOLUTIONS

The mathematical formulation of modified gravity theories begins with a generalization of Einstein's field equations. In General Relativity (GR), the dynamics of spacetime are governed by the Einstein-Hilbert action, which is given by:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R + S_m,$$

where R is the Ricci scalar, gg is the determinant of the metric tensor gμν, κ2=8πG and Sm is the action for matter fields. Varying this action with respect to the metric yields Einstein's field equations:

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu},$$

where Gμν is the Einstein tensor and Tμν is the energy-momentum tensor.

In modified gravity theories, this action is altered to allow for additional geometric or field contributions. One of the simplest extensions is f(R) gravity, where the action becomes:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m.$$

The variation of this action leads to fourth-order differential equations of motion, rather than the second-order equations in GR. These modified field equations are:

$$f_R R_{\mu\nu} - \frac{1}{2} f(R) g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R = \kappa^2 T_{\mu\nu},$$

where $f_R = \frac{df}{dR}$, $\Box$ is the d'Alembertian operator, and ∇_μ denotes covariant differentiation. These equations introduce new dynamics to the universe's expansion, allowing for self-acceleration solutions under appropriate forms of $f(R)$, such as $f(R) = R + \alpha R^n$ or $f(R) = R - \mu^4/R$.

In the context of scalar-tensor theories, such as the Brans–Dicke model or general Horndeski theory, the action includes a scalar field ϕϕ that couples to gravity:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} F(\phi) R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] + S_m.$$

The resulting field equations include both a modified Einstein equation and a scalar field equation. The scalar field influences the gravitational strength dynamically, allowing for variable cosmological behaviour. When analysed in a Friedmann-Lemaître-Robertson-Walker (FLRW) background metric, these theories yield modified Friedmann equations that describe the expansion of the universe.

Similarly, in f(T) gravity, the action is based on the torsion scalar T, derived from the Weitzenböck connection rather than the Levi-Civita connection used in GR. The action takes the form:

$$S = \frac{1}{2\kappa^2} \int d^4x \, e \, f(T) + S_m,$$

where $e = \det(e^A_\mu)$ is the determinant of the tetrad field. The field equations derived from this action are second-order, like in GR, which makes $f(T)$ gravity easier to handle mathematically than $f(R)$ gravity. When applied to the FLRW metric, the modified Friedmann equations in $f(T)$ gravity take the form:

$$H^2 = \frac{\kappa^2}{3}\rho - \frac{1}{6}(f(T) - 2T f_T),$$

where $f_T = \frac{df}{dT}$, H is the Hubble parameter, and ρ is the total energy density. Suitable choices of $f(T)$, such as $f(T) = T + \beta T^n$, can account for the observed acceleration without a cosmological constant.

Each of these theories modifies the standard Friedmann equations, which in General Relativity are:

$$H^2 = \frac{\kappa^2}{3}\rho, \quad \dot{H} = -\frac{\kappa^2}{2}(\rho + p),$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, ρ is the energy density, and p is the pressure. In modified theories, additional terms from curvature functions, scalar fields, or torsion contribute to the dynamics, effectively acting as a time-varying cosmological term.

These modified cosmological solutions allow for the modelling of a universe that transitions from a matter-dominated decelerating phase to a late-time accelerating phase. By choosing appropriate functions or parameters in the action, one can recover observationally consistent expansion histories. The flexibility in these models enables them to mimic dark energy behaviour and potentially solve the fine-tuning problems associated with the cosmological constant.

In summary, the mathematical structure of modified gravity theories introduces extra degrees of freedom that naturally lead to late-time cosmic acceleration. The corresponding cosmological solutions are rich and varied, capable of describing a universe that aligns with current observational data, while also offering new insights into the nature of gravitational interaction.

5. OBSERVATIONAL CONSTRAINTS ON MODIFIED GRAVITY

Theoretical elegance alone is not sufficient for the acceptance

of modified gravity theories; they must also conform to a wide array of high-precision astronomical observations. Over the past two decades, a wealth of observational data has been collected that helps constrain the parameters and viability of these models. Observational constraints play a crucial role in distinguishing modified gravity theories from the standard ΛCDM model and from each other. These constraints come from multiple cosmological probes, including Type Ia supernovae, Cosmic Microwave Background (CMB) anisotropies, Baryon Acoustic Oscillations (BAO), large-scale structure (LSS), gravitational lensing, and more recently, gravitational wave observations.

One of the earliest and most significant constraints on cosmic acceleration came from the Type Ia supernovae observations. These standard candles allowed for the construction of the Hubble diagram, providing a direct measurement of the expansion history of the universe. Modified gravity theories must reproduce the observed luminosity-distance relation in order to remain viable. For instance, in f(R) gravity models, the function f(R) must be tuned such that the predicted scale factor evolution matches the observed redshift-distance data.

The Cosmic Microwave Background (CMB), particularly as measured by the WMAP and Planck satellites, imposes stringent constraints on any deviations from General Relativity. The CMB encodes information about the early universe and its evolution through the angular power spectrum. Modified gravity can affect the growth rate of perturbations, the integrated Sachs-Wolfe (ISW) effect, and the lensing potential. For example, scalar-tensor theories can alter the evolution of gravitational potentials, thus changing the ISW effect, which is sensitive to changes in gravitational dynamics. As such, CMB measurements are used to place bounds on the scalar field dynamics and coupling functions in these theories.

Baryon Acoustic Oscillations (BAO) provide another powerful tool to constrain the expansion history. As a standard ruler, the BAO scale can be used to test whether the background expansion predicted by modified gravity matches observations. The consistency of BAO observations with the ΛCDM model creates a high benchmark for any alternative theory to meet. Any modified gravity theory must carefully adjust its parameters to ensure that it does not distort the sound horizon scale or produce inconsistent results with redshift-space distortions.

Large-scale structure (LSS) surveys track the distribution and clustering of galaxies over cosmic time, offering insights into the growth of matter density perturbations. Since modified gravity affects both the background evolution and the growth of structure, measurements of the growth rate $f = d\ln\delta/d\ln a$ and its product with the matter power spectrum amplitude, $f\sigma 8$, are essential observational tests. Some models of f(R) gravity predict enhanced growth rates due to the additional scalar degrees of freedom, which can be constrained through redshift-space distortion measurements and galaxy surveys like the Sloan Digital Sky Survey (SDSS) and the Dark Energy Survey (DES).

Weak gravitational lensing—the deflection of light by intervening matter—provides yet another independent test of gravity on cosmic scales. Modified gravity can influence the lensing potential differently from General Relativity, particularly if it modifies the relationship between the Newtonian potentials Φ and Ψ. Observational results from the Kilo-Degree Survey (KiDS), Hyper Suprime-Cam (HSC), and others can be used to reconstruct the matter power spectrum and test for deviations from GR, offering strong constraints on models that modify the Poisson equation or introduce anisotropic stresses.

Recently, gravitational wave observations, especially those from binary neutron star mergers like GW170817, have provided a novel constraint: the speed of gravitational waves must be extremely close to the speed of light. This result severely constrains a wide class of scalar-tensor theories, including those with non-minimal kinetic couplings or higher-order derivatives, ruling out many previously viable models. The tight bound on the difference between the speed of light and gravitational waves forces many modified gravity theories to reconfigure their parameter space or abandon certain couplings altogether.

Overall, while several modified gravity theories can reproduce the late-time acceleration of the universe, only those that pass the rigorous observational tests across multiple data sets are considered viable. Current observations, though consistent with the ΛCDM model, do not completely exclude modified gravity theories—some remain viable within specific parameter ranges and under particular assumptions. However, future surveys such as Euclid, the Vera C. Rubin Observatory (LSST), and the Square Kilometre Array (SKA) are expected to increase the precision of cosmological measurements dramatically. These missions will further constrain, or potentially uncover, deviations from General Relativity, offering a more definitive test of the modified gravity hypothesis.

6. CASE STUDIES AND COMPARATIVE ANALYSIS

To evaluate the viability and explanatory power of modified gravity theories, researchers often conduct detailed case studies involving specific models. These case studies provide insight into how well each model accounts for observed phenomena such as the universe's expansion history, structure formation, and gravitational dynamics. Through comparative analysis, it is possible to discern which models offer the best theoretical and observational fit, and which encounter significant shortcomings.

One of the most frequently studied examples in this context is the Hu–Sawicki model of f(R) gravity, proposed to closely mimic the expansion history of the ΛCDM model while introducing minimal deviations from General Relativity at high curvatures. This model is defined by a functional form of f(R) that includes parameters tuned to replicate late-time acceleration. Case studies involving this model show that it can produce a viable cosmic expansion history, pass solar system tests via chameleon screening mechanisms, and provide distinctive signatures in the growth of structure that can be tested with large-scale galaxy surveys. Comparative studies against ΛCDM demonstrate that although Hu–Sawicki gravity performs similarly at the background level, it predicts enhanced clustering at late times,

which may be detectable with future data.

Another compelling example is the Starobinsky model of f(R) gravity, which adds a term proportional to R2 to the Einstein-Hilbert action. Originally developed to explain inflation, this model has been extended to study late-time acceleration. Although it predicts viable cosmological evolution, some versions require fine-tuning to meet local gravity constraints. In comparative analyses, the Starobinsky model tends to yield slightly different predictions for the matter power spectrum and weak lensing shear correlations, which can be tested through high-precision cosmic shear data.

Brans–Dicke theory, one of the earliest scalar-tensor theories, offers a case study in the dynamic variation of the gravitational "constant." With a coupling parameter ω, the theory reduces to General Relativity in the limit ω→∞. Observationally, solar system experiments have already placed very strong constraints on ω, typically requiring ω>40,000 limiting the extent to which the theory can differ from General Relativity. While some cosmological models based on Brans–Dicke theory can accommodate late-time acceleration, they often require additional potentials or tuning, making them less appealing compared to newer scalar-tensor models like those in the Horndeski class.

A distinctive branch of modified gravity is teleparallel gravity, particularly the f(T) models. In these theories, torsion, rather than curvature, is the key geometric quantity governing gravity. Case studies involving simple extensions such as f(T)=T+αTn demonstrate that such models can give rise to acceleration without dark energy. Unlike f(R) gravity, the field equations remain second-order, simplifying their cosmological application. However, when subjected to observational comparison—particularly with structure formation and weak lensing—f(T) models sometimes struggle to match ΛCDM-level accuracy unless their parameters are tightly constrained.

Comparative analysis also includes DGP braneworld models, where the leakage of gravity into extra dimensions at large scales provides a mechanism for self-acceleration. While this model can produce late-time acceleration without dark energy, it suffers from ghost instabilities in its self-accelerating branch and faces tension with structure formation data and CMB constraints. Modifications such as including a scalar field (e.g., Galileons) have been proposed to overcome these issues, but they too face strict bounds from the speed of gravitational waves and stability conditions.

Overall, comparative studies reveal that no single modified gravity theory outperforms the ΛCDM model across all tests. Most alternatives either approximate ΛCDM at the background level or exhibit distinctive features in structure growth or gravitational lensing that allow them to be constrained or falsified. Importantly, the combination of multiple probes—such as supernovae, CMB, BAO, lensing, and redshift-space distortions—provides a powerful framework for distinguishing between models.

In summary, case studies and comparative analyses of different modified gravity theories highlight their strengths, limitations, and observational viability. While several models offer compelling alternatives to dark energy, especially in explaining cosmic acceleration, they must navigate a narrow parameter space constrained by a multitude of observations. The ongoing and upcoming cosmological surveys are expected to further tighten these constraints, enabling an even clearer comparison and possibly pointing toward the correct extension—or confirmation—of Einstein's gravitational theory.

7. CONCLUSION

The study of the expansion of the universe through the lens of modified gravity theories represents a bold and intellectually rich endeavour within modern cosmology. As the late-time accelerated expansion of the universe remains one of the most profound mysteries in physics, the limitations of the standard ΛCDM model—particularly its reliance on an enigmatic and finely tuned cosmological constant—have led to a search for alternative explanations. Modified gravity theories seek to explain cosmic acceleration not by invoking a new energy component, but by reconsidering and extending the fundamental laws of gravity themselves.

Through a range of theoretical frameworks—including f(R) gravity, scalar-tensor theories, f(T) gravity, and braneworld models—researchers have developed models capable of reproducing the observed expansion history and, in some cases, offering testable deviations from General Relativity. These models provide richer dynamics and introduce new degrees of freedom that can mimic the effects of dark energy or offer entirely new perspectives on cosmic acceleration.

However, theoretical promise must be matched by empirical validity. Modified gravity theories are subject to rigorous scrutiny through multiple observational probes, such as Type Ia supernovae, the Cosmic Microwave Background, Baryon Acoustic Oscillations, large-scale structure formation, and gravitational lensing. Recent constraints from gravitational wave astronomy have further narrowed the parameter space of many scalar-tensor models. While some modified gravity theories remain viable within specific ranges and under well-defined conditions, many face serious challenges when confronted with the full spectrum of observational data.

Comparative case studies illustrate that while modified gravity theories can replicate the background expansion seen in the ΛCDM model, their predictions often diverge at the level of perturbations and structure formation—differences that high-precision cosmological surveys are increasingly capable of detecting. The Hu–Sawicki and Starobinsky f(R) models, certain Horndeski-type scalar-tensor theories, and constrained f(T) models continue to be actively studied as potentially realistic alternatives.

Overall, modified gravity remains a compelling approach to addressing the puzzle of cosmic acceleration. While the ΛCDM model continues to provide an excellent empirical fit, it does so at the cost of introducing a cosmological constant with

unclear physical origin. Modified gravity theories, by contrast, aim for a deeper understanding of gravitational dynamics that could unify cosmic acceleration with the broader fabric of spacetime physics. As future surveys and observational programs deliver unprecedented levels of precision, we move closer to determining whether cosmic acceleration is a sign of new physics in the energy content of the universe—or a hint that our understanding of gravity itself is incomplete.

REFRENCES

1. Bahamonde, S., Böhmer, C. G., & Wright, M. (2015). Modified teleparallel theories of gravity. Physical review D, 92(10), 104042.
2. Koyama, K. (2016). Cosmological tests of modified gravity. Reports on Progress in Physics, 79(4), 046902.
3. Lobo, F. S. (2008). The dark side of gravity: Modified theories of gravity. arXiv preprint arXiv:0807.1640.
4. Myrzakulov, R., Sebastiani, L., & Zerbini, S. (2013). Some aspects of generalized modified gravity models. International Journal of Modern Physics D, 22(08), 1330017.
5. Olmo, G. J., Rubiera-Garcia, D., & Wojnar, A. (2020). Stellar structure models in modified theories of gravity: Lessons and challenges. Physics Reports, 876, 1-75.
6. Paul, B. C., Debnath, P. S., & Ghose, S. (2009). Accelerating universe in modified theories of gravity. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 79(8), 083534.
7. Petrov, A., Nascimento, J. R., & Porfirio, P. (2023). Introduction to modified gravity. Springer Nature.
8. Sbisà, F. (2014). Modified theories of gravity. arXiv preprint arXiv:1406.3384.
9. Seifert, M. D. (2007). Stability of spherically symmetric solutions in modified theories of gravity. Physical Review D—Particles, Fields, Gravitation, and Cosmology, 76(6), 064002.
10. Shankaranarayanan, S., & Johnson, J. P. (2022). Modified theories of gravity: Why, how and what?. General Relativity and Gravitation, 54(5), 44.